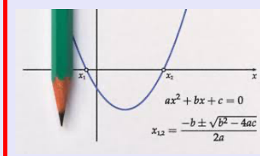
Math 125

Spring 2022

Lecture 18



Class QZ 15:

Solve

$$\begin{cases} x^2 + \cancel{y^2} = 13 \\ x^2 - \cancel{y^2} = 5 \end{cases}$$

$$\frac{2x^2}{2} = \frac{18}{2}$$

$$x^2 = 9$$

$$\boxed{x = \pm 3}$$

$$9 + y^2 = 13$$

$$y^2 = 4$$

$$\boxed{y = \pm 2}$$

$$\boxed{\text{final Ans: } \{(3,2),(3,-2),(-3,2),(-3,-2)\}}$$

Back to radicals

Given $f(x) = \sqrt{6x+1}$

1) Find $f(0)$

$f(0) = \sqrt{6(0)+1}$

$= \sqrt{0+1} = \sqrt{1} = \boxed{1}$

2) Find $f(4)$

$f(4) = \sqrt{6(4)+1}$

$= \sqrt{24+1} = \sqrt{25} = \boxed{5}$

3) Find the domain of $f(x)$.

$f(x) = \sqrt{6x+1}$

even root function $\Rightarrow$ Radicand ≥ 0

$6x+1 \geq 0 \quad 6x \geq -1$

$x \geq \frac{-1}{6} \Rightarrow \boxed{\left[\frac{-1}{6}, \infty\right)}$

Simplify

1) $\sqrt{3}\sqrt{6} = \sqrt{18} = \sqrt{9}\sqrt{2} = \boxed{3\sqrt{2}}$

2) $\sqrt{5}\sqrt{20} = \sqrt{100} = \boxed{10}$

Index=2

3) $\sqrt{20} + \sqrt{5} = \sqrt{4}\sqrt{5} + \sqrt{5} = 2\sqrt{5} + 1\sqrt{5}$

$= \boxed{3\sqrt{5}}$

4) $\sqrt{18} - \sqrt{2} = \sqrt{9}\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 1\sqrt{2}$

$= \boxed{2\sqrt{2}}$

Assume $x \geq 0$

$\sqrt[2]{x^2} = x$

$\sqrt[3]{x^3} = x$ $\Rightarrow$ $\boxed{\sqrt[n]{x^n} = x}$ why? (Answer)$^{\text{Index}}$ = Radicand

$\sqrt[4]{x^4} = x$

Simplify:

1) $\sqrt[\cancel{4}]{(x+2)^{\cancel{4}}}$

$= \boxed{x+2}$

2) $\sqrt[\cancel{7}]{(x-3)^{\cancel{7}}}$

$= \boxed{x-3}$

Recall $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

1) write as rational exponent:

$\sqrt[4]{x^3} = x^{\frac{3}{4}}$

$\sqrt[2]{x^1} = x^{\frac{1}{2}}$

$\sqrt[5]{x^2} = x^{\frac{2}{5}}$

2) write in radical notation

$x^{\frac{1}{3}} = \sqrt[3]{x^1} = \sqrt[3]{x}$

$(x-2)^{\frac{2}{5}} = \sqrt[5]{(x-2)^2}$

$(2x+1)^{\frac{1}{2}} = \sqrt[2]{(2x+1)^1} = \sqrt{2x+1}$

Write as a single radical:

1) $\sqrt[3]{x^2} \cdot \sqrt[4]{x} = x^{\frac{2}{3}} \cdot x^{\frac{1}{4}} = x^{\frac{2}{3}+\frac{1}{4}} = x^{\frac{11}{12}}$

$= \boxed{\sqrt[12]{x^{11}}}$

2) $\dfrac{\sqrt[5]{x^2}}{\sqrt[3]{x}} = \dfrac{x^{\frac{2}{5}}}{x^{1/3}} = x^{\frac{2}{5}-\frac{1}{3}}$

$= x^{\frac{1}{15}} = \sqrt[15]{x^1} = \boxed{\sqrt[15]{x}}$

3) $\sqrt[6]{\sqrt[4]{x^3}} = \left(\sqrt[4]{x^3}\right)^{\frac{1}{6}} = \left(x^{\frac{3}{4}}\right)^{\frac{1}{6}} = x^{\frac{\cancel{3}}{4} \cdot \frac{1}{\cancel{6}_2}}$

$= x^{\frac{1}{8}}$

$= \sqrt[8]{x^1}$

$= \boxed{\sqrt[8]{x}}$

Simplify

1) $\sqrt[10]{x^2} = x^{\frac{2}{10}} = x^{\frac{1}{5}} = \sqrt[5]{x^1} = \boxed{\sqrt[5]{x}}$

2) $\sqrt[12]{x^3} = x^{\frac{3}{12}} = x^{\frac{1}{4}} = \sqrt[4]{x^1} = \boxed{\sqrt[4]{x}}$

3) $\sqrt[15]{x^{10}} = x^{\frac{10}{15}} = x^{\frac{2}{3}} = \boxed{\sqrt[3]{x^2}}$

4) $\sqrt[5]{x^2} \cdot \sqrt[5]{x^3} = x^{\frac{2}{5}} \cdot x^{\frac{3}{5}} = x^{\frac{2}{5}+\frac{3}{5}} = x^{\frac{5}{5}} = x^1 = \boxed{x}$

5) $\dfrac{\sqrt[12]{x^7}}{\sqrt[4]{x}} = \dfrac{x^{\frac{7}{12}}}{x^{1/4}} = x^{\frac{7}{12}-\frac{1}{4}} = x^{\frac{4}{12}} = x^{\frac{1}{3}} = \boxed{\sqrt[3]{x}}$

Simplify

1) $\sqrt{300} = \sqrt{100}\sqrt{3} = \boxed{10\sqrt{3}}$

2) $\sqrt{10}\sqrt{5} = \sqrt{50} = \sqrt{25}\sqrt{2} = \boxed{5\sqrt{2}}$

3) $\sqrt{80} - \sqrt{20} = \sqrt{16}\sqrt{5} - \sqrt{4}\sqrt{5}$

$= 4\sqrt{5} - 2\sqrt{5} = \boxed{2\sqrt{5}}$

Distribute/Foil, then Simplify:

1) $\sqrt{6}(\sqrt{3} - \sqrt{6}) = \sqrt{18} - \sqrt{36}$

$= \sqrt{9}\sqrt{2} - 6 = \boxed{3\sqrt{2} - 6}$

2) $(2\sqrt{2} + 3)(3\sqrt{2} + 1)$

$= 6\sqrt{4} + 2\sqrt{2} + 9\sqrt{2} + 3$

$= 6\cdot 2 + 11\sqrt{2} + 3 = \boxed{15 + 11\sqrt{2}}$

3) $(6\sqrt{5} + 3\sqrt{2})(2\sqrt{5} - 4\sqrt{2})$

$= 12\sqrt{25} \quad - 24\sqrt{10} + 6\sqrt{10} \quad - 12\sqrt{4}$

$= 12 \cdot 5 \quad -18\sqrt{10} \quad -12 \cdot 2$

$= 60 - 18\sqrt{10} - 24 = \boxed{36 - 18\sqrt{10}}$

4) $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})$

$= \sqrt{49} - \cancel{\sqrt{21}} \cancel{+ \sqrt{21}} - \sqrt{9}$

$= 7 \qquad -3 = \boxed{4}$

5) $(3\sqrt{2} + 1)^2$

Hint: $x^2 = x \cdot x$

$= (3\sqrt{2} + 1)(3\sqrt{2} + 1)$

$= 9\sqrt{4} + 3\sqrt{2} + 3\sqrt{2} + 1 = 9 \cdot 2 + 6\sqrt{2} + 1$

$= \boxed{19 + 6\sqrt{2}}$

6) $(2\sqrt{3} - 1)^2$

$= (2\sqrt{3} - 1)(2\sqrt{3} - 1)$

$= 4\sqrt{9} - 2\sqrt{3} - 2\sqrt{3} + 1$

$= 4 \cdot 3 - 4\sqrt{3} + 1 = \boxed{13 - 4\sqrt{3}}$

$(\sqrt{x})^2 = x$

$(\sqrt[3]{x})^3 = x \quad \Rightarrow (\sqrt[n]{x})^n = x$

$(\sqrt[4]{x})^4 = x$

Solve Simple radical equations:

1) Isolate the radical

2) Raise both Sides to the index power

3) Simplify and Solve

4) **Always check all Solutions in the original equation.**

Solve $\sqrt{x-2} - 3 = 0$

$\sqrt{x-2} = 3$

$(\sqrt{x-2})^2 = (3)^2$

$x - 2 = 9$

$\boxed{x = 11}$

Check

$\sqrt{11-2} - 3 = 0$

$\sqrt{9} - 3 = 0$

$3 - 3 = 0$

$0 = 0$ ✓

$\{11\}$

Solve $\sqrt[3]{x+1} + 2 = 0$

$\sqrt[3]{x+1} = -2$

$(\sqrt[3]{x+1})^3 = (-2)^3$

$x + 1 = -8$

$\boxed{x = -9}$

$\{-9\}$

Check:

$\sqrt[3]{-9+1} + 2 = 0$

$\sqrt[3]{-8} + 2 = 0$

$-2 + 2 = 0$

$0 = 0$ ✓

Solve $\sqrt{2x-1}+5=2$

Index=2

$\sqrt{2x-1}=2-5$

$\boxed{\sqrt{2x-1}=-3}$

$(\sqrt{2x-1})^2=(-3)^2$

$2x-1=9$

$2x=10$

$\boxed{x=5}$ ← Extraneous Solution

No other Solutions

even root

Ans $-3<0$

$\emptyset$

Check:

$\sqrt{2(5)-1}+5=2$

$\sqrt{10-1}+5=2$

$\sqrt{9}+5=2$

$3+5=2$

$8=2$

False

No Solution

$\emptyset$, $\{\ \}$

Solve $\sqrt{6x+7}-x=2$

Index=2

$\sqrt{6x+7}=x+2$

$(\sqrt{6x+7})^2=(x+2)^2$

$6x+7=(x+2)(x+2)$ Foil & Simplify

$6x+7=x^2+4x+4$

Make one Side Zero,

$x^2+4x+4-6x-7=0$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$x+1=0$ $\qquad$ $x-3=0$

$\boxed{x=-1}$ $\qquad$ $\boxed{x=3}$

$\{-1,3\}$

Check $x=-1$

$\sqrt{6(-1)+7}-(-1)=2$

$\sqrt{-6+7}+1=2$

$\sqrt{1}+1=2$

$1+1=2$

$2=2$ ✓

Check $x=3$

$\sqrt{6(3)+7}-3=2$

$\sqrt{25}-3=2$

$5-3=2$

$2=2$ ✓

Solve $x - \sqrt{6x+7} = 0$

$x = \sqrt{6x+7}$

$(x)^2 = (\sqrt{6x+7})^2$

$x^2 = 6x + 7$

$x^2 - 6x - 7 = 0$

$(x+1)(x-7) = 0$

$x+1=0$ $\quad$ $x-7=0$

$x=-1$ $\quad$ $x=7$

$\{7\}$

Check

$x=-1$ ← E.S.

$-1 - \sqrt{6(-1)+7} = 0$

$-1 - \sqrt{1} = 0$

$-1 - 1 = 0$

$-2 = 0$

false

$\boxed{x=7}$ ✓

$7 - \sqrt{6(7)+7} = 0$

$7 - \sqrt{42+7} = 0$

$7 - \sqrt{49} = 0$

$7 - 7 = 0$

$0 = 0$ ✓

Solve $x = \sqrt{3x+7} - 3$

$x + 3 = \sqrt{3x+7}$

$(x+3)^2 = (\sqrt{3x+7})^2$

Foil & Simplify $(x+3)(x+3) = 3x+7$

$x^2 + 6x + 9 = 3x + 7$

$x^2 + 6x + 9 - 3x - 7 = 0$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x+2=0$ $\quad$ $x+1=0$

$\boxed{x=-2}$ $\quad$ $\boxed{x=-1}$

Check

$x=-2$

$-2 = \sqrt{3(-2)+7} - 3$

$-2 = \sqrt{1} - 3$

$-2 = 1 - 3$

$-2 = -2$ ✓

$-1 = \sqrt{3(-1)+7} - 3$

$-1 = \sqrt{4} - 3$

$-1 = 2 - 3$

$-1 = -1$ ✓

$\{-2, -1\}$

Conjugate Expressions:

$a+b$ & $a-b$ are conjugates.

Multiply $2\sqrt{2}+3$ by its conjugate.

$(2\sqrt{2}+3)(2\sqrt{2}-3)=$

$4\sqrt{4}\cancel{-6\sqrt{2}}\cancel{+6\sqrt{2}}-9=$

$4\cdot 2-9=8-9=\boxed{-1}$

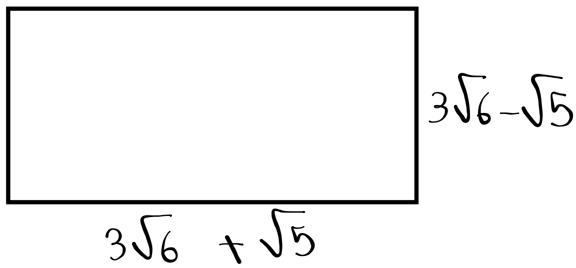
Multiply $2\sqrt{5}-\sqrt{3}$ by its conjugate.

$(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})$

$=4\sqrt{25}\cancel{+2\sqrt{15}}\cancel{-2\sqrt{15}}-\sqrt{9}$

$=4\cdot 5-3=\boxed{17}$

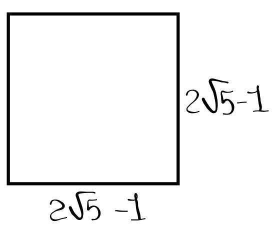
Find Area & Perimeter:



$$P = 2L + 2W$$
$$= 2(3\sqrt{6}+\sqrt{5}) + 2(3\sqrt{6}\cdot\sqrt{5})$$
$$= 6\sqrt{6} + 2\sqrt{5} + 6\sqrt{6} - 2\sqrt{5}$$
$$= \boxed{12\sqrt{6}}$$

$$A = LW = (3\sqrt{6}+\sqrt{5})(3\sqrt{6}-\sqrt{5})$$
$$= 9\sqrt{36} \cancel{-3\sqrt{30}} \cancel{+3\sqrt{30}} - \sqrt{25}$$
$$= 9\cdot 6 - 5 = \boxed{49}$$

Find perimeter and area:



$$P = 4S$$
$$= 4(2\sqrt{5}-1)$$
$$= \boxed{8\sqrt{5}-4}$$

$$A = S^2$$
$$= (2\sqrt{5}-1)^2$$
$$= (2\sqrt{5}-1)(2\sqrt{5}-1)$$
$$= 4\sqrt{25} - 2\sqrt{5} - 2\sqrt{5} + 1$$
$$= (4\cdot 5) - 4\sqrt{5}\ (+1)$$

$$\text{Area} = \boxed{21 - 4\sqrt{5}}$$